CEE Growth & Development

Solow Model

Michælmas Term 2013

Solow model basics

- Cobb-Douglas Production Function

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$$

- Fundamental Law of Motion

$$\Delta K_t = s \cdot Y_t - \delta \cdot K_t$$

  - when $L_t = L_{t-1} = \bar{L}$ and $x_t = \frac{X_t}{L_t}$

$$\Delta k_t = s \cdot A k_t^{\alpha} - \delta \cdot k_t$$

- Steady state

$$\Delta K_t = 0 \left[= \frac{\Delta K_t}{K_t} = g_K = \frac{\dot{K}}{K} \right]$$

Steady State

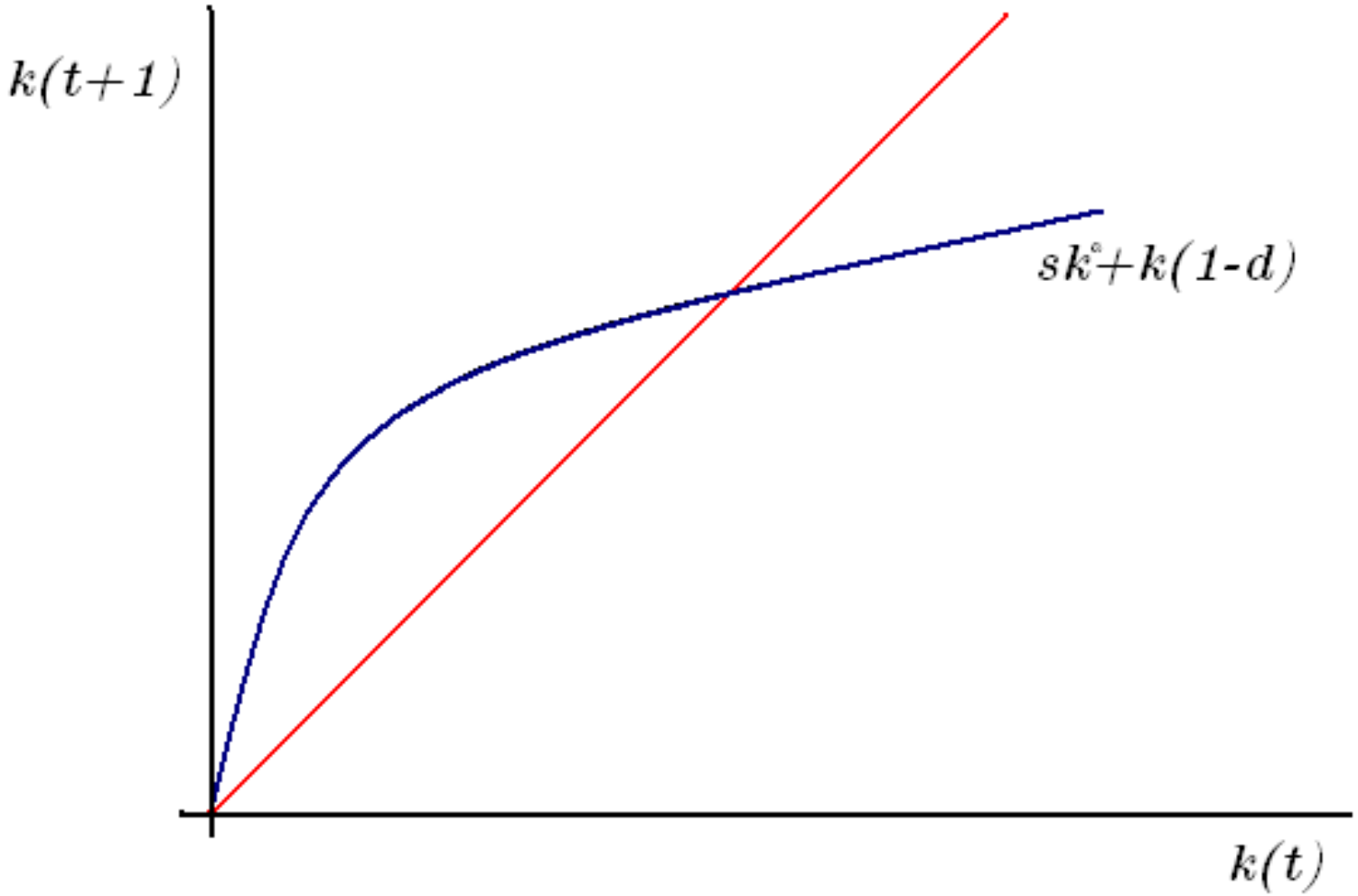
- Assuming $L_t = \bar{L}$

$$\Delta K_t = 0 \overset{L_t=\bar{L}}{\Longrightarrow} \Delta k_t = \frac{\Delta k_t}{k_t} = g_k = 0$$

- From the production function:

$$\begin{aligned} \Delta K_t &= 0 \overset{\substack{Y_t=AK_t^{\alpha}L_t^{1-\alpha} \\ L_t=\bar{L}}}{\Longrightarrow} \Delta Y_t = \frac{\Delta Y_t}{Y_t} = g_Y = 0 \\ \Delta y_t &= \frac{\Delta y_t}{y_t} = \frac{\dot{y}}{y} = g_y = 0 \end{aligned}$$

Solow Diagram 2



Solow Steady State

$$\Delta k_t = s \cdot A k_t^{\alpha} - \delta \cdot k_t$$

and

$$\frac{\Delta k_t}{k_t} = 0$$

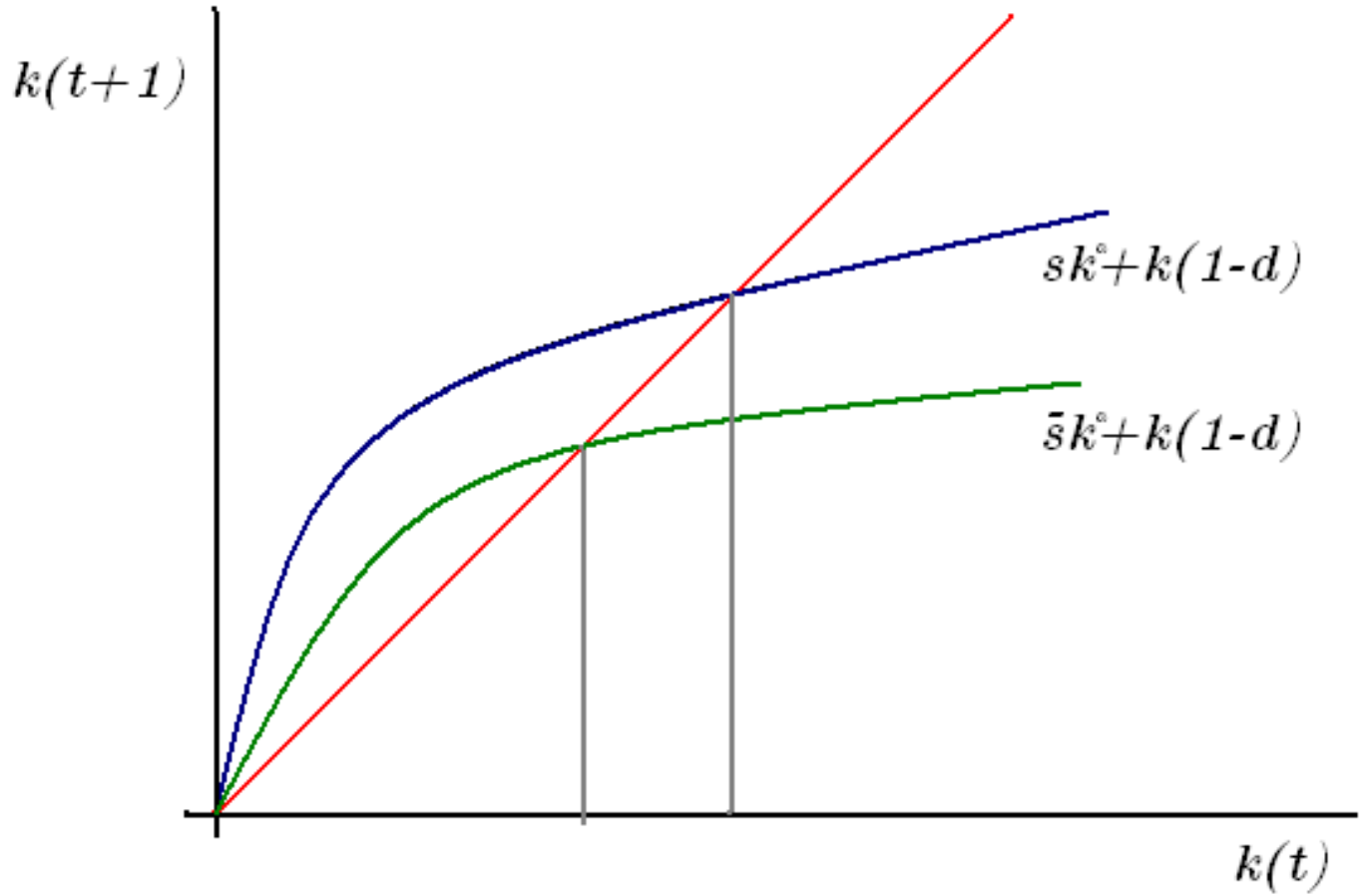
gives

$$s \cdot A k_t^{\alpha - 1} = \delta$$

$$k^* = \left[\frac{s \cdot A}{\delta} \right]^{\frac{1}{1-\alpha}}$$

$$y^* = A \cdot \left[\frac{s \cdot A}{\delta} \right]^{\frac{\alpha}{1-\alpha}}$$

Solow Diagram: Reform (Different Savings Rate)

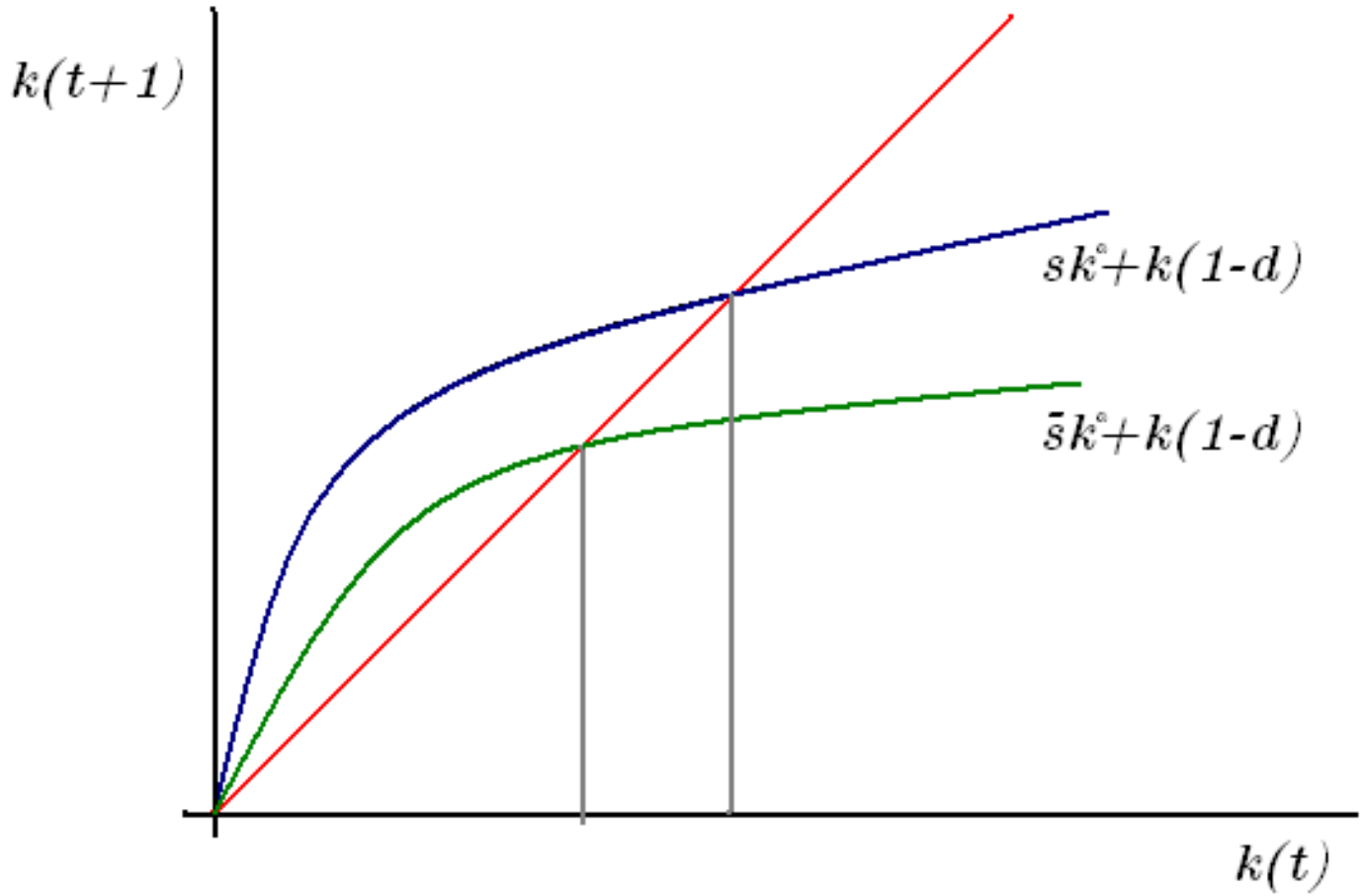


Different Savings Rate and Level Effect

$$k^* = \left[\frac{s \cdot A}{\delta}\right]^{\frac{1}{1-\alpha}} \text{ and } k^{**} = \left[\frac{s' \cdot A}{\delta}\right]^{\frac{1}{1-\alpha}}$$

$$y^* = A \cdot \left[\frac{s \cdot A}{\delta}\right]^{\frac{\alpha}{1-\alpha}} \text{ and } y^{**} = A \cdot \left[\frac{s' \cdot A}{\delta}\right]^{\frac{\alpha}{1-\alpha}}$$

Increased Savings Rate and Holodomor



'Golden Rule' Savings Rate

$$\begin{aligned} \max_{s} c &= y - sy \\ & s.t. \\ y &= Ak^{\alpha} \\ k &= \left[\frac{s \cdot A}{\delta}\right]^{\frac{1}{1-\alpha}} \end{aligned}$$

Solow Model Predictions

- Countries will eventually end up in their steady states
- Countries will grow at different rates if they are at different distance from the steady state level
    - in case they have the same steady state

Stability, Multiple Equlibria and Poverty trap